

Nonlocal Phenomena from Noncommutative Pre-Planckian Regime

Michael Heller*

Vatican Observatory,
V-00120 Vatican City State

Wiesław Sasin

Institute of Mathematics,
Warsaw University of Technology,
Plac Politechniki 1, 00-661 Warsaw, Poland

February 7, 2008

Abstract

A model unifying general relativity with quantum mechanics is further developed. It is based on a noncommutative geometry which supposedly modelled the universe in its pre-Planckian epoch. The geometry is totally nonlocal with no time and no space in their usual meaning. They emerge only in the transition process from the noncommutative epoch to the standard space-time physics. Observational aspects of this model are discussed. It is shown that various nonlocal phenomena can be explained as remnants of the primordial noncommutative epoch. In particular, we explain the Einstein-Podolsky-Rosen experiment, the horizon problem in cosmology, and the appearance of singularities in general relativity.

*Correspondence address: ul. Powstańców Warszawy 13/94, 33-110 Tarnów, Poland,
e-mail: mheller@wsd.tarnow.pl

1 Introduction

There are many hints coming from several independent research programs that in the quantum gravity regime the standard concepts of space and time break down. Following this suggestion we propose a scheme for unification of general relativity and quantum mechanics based on a radically nonlocal geometry in which such structures as space point, time instant, and their neighborhoods, cannot be even defined. Namely, we assume that it is a noncommutative geometry (see [10] for an introductory course and [1] for a comprehensive monograph) that correctly models the universe below the Planck threshold, and only during the “phase transition” from a noncommutative geometry to the standard (commutative) geometry, space, time and other local structures emerge. In [9] we have proposed a concrete model implementing the above ideology. Although based on a sound mathematical basis it should be treated as a toy model (because of some both conceptual and computational simplifications). It turns out that some nonlocal phenomena, known from quantum mechanics and cosmology, such as the Einstein-Podolsky-Rosen (EPR) type of experiments, the horizon problem, and the appearance of classical singularities, can be explained as remnants (or “shadows”) of the primordial global physics. To discuss these phenomena in terms of a noncommutative geometry is the goal of the present paper. Although the analysis is based on our model presented in [9], it is independent of the particulars of this model. In section 2, we give a brief review of our model, and in sections 3 through 5 we discuss, in turn, the EPR experiment, the horizon problem, and the classical singularity problem.

2 An Overview of the Model

Let $\pi_M : E \rightarrow M$ be a fiber bundle of orthonormal frames over a space-time manifold M , and Γ a group. In the following, for concreteness, we shall assume that $\Gamma = \text{SO}(3,1)$, but the correct choice of Γ should be based on physical grounds and is left for the future developments of the model. The Cartesian product $G = E \times \Gamma$ has the structure of the groupoid (unlike in a group its elements can be composed only if they belong to the same fiber). We define the involutive algebra \mathcal{A} of smooth, complex valued, compactly

supported functions on G with the convolution

$$(a * b)(\gamma) := \int_{G_p} a(\gamma_1) b(\gamma_2)$$

as multiplication, where $a, b \in \mathcal{A}$, $\gamma = \gamma_1 \circ \gamma_2$, $\gamma, \gamma_1, \gamma_2 \in G_p$, G_p is the fiber in G over $p \in E$, and the integral is taken with respect to the (left) invariant Haar measure. The involution in \mathcal{A} is defined in the following way: $a^*(\gamma) := \overline{a(\gamma^{-1})}$. The noncommutative algebra \mathcal{A} is an analogue of the algebra of smooth functions on a manifold and serves us to construct a noncommutative geometry of our model.

Let V be a subset of derivations of \mathcal{A} . A derivation $v \in V$ is defined to be a linear mapping $v : \mathcal{A} \rightarrow \mathcal{A}$ satisfying the Leibniz rule. It can be interpreted as a counterpart of a vector field. We assume that V is of the form $V = V_E \oplus V_\Gamma$ where V_E are derivations “parallel to E ” and V_Γ derivations “parallel to Γ ”. Basing on V one can define all fundamental concepts of differential geometry, such as: linear connection, curvature, Ricci tensor and, consequently, one can write a noncommutative generalization of Einstein’s equation. In the natural way, this geometry splits into the part “parallel to E ” and the part “parallel to Γ ” (for details see [8, 9] and the bibliography cited therein).

The special care should be given to the metric problem. It has been showed by Madore and Mourad [11] that for a broad class of derivation based differential geometries the metric is essentially unique. It turns out that in our case this applies to the Γ -parallel part of the metric g_Γ , whereas the E -parallel part of the metric g_E behaves in the standard way (it is essentially lifting of the metric from space-time M). Einstein’s equation surprisingly well cooperates with this fact. It is an operator equation which should be solved for derivations (not for a metric!). However, since all derivations belonging to V_E solve the E -parallel part of Einstein’s equation this part of the equation becomes an equation for metric (for details see [8]).

Let G_p and G_q be two fibres of G over $p, q \in E$, respectively. This fibres are said to be equivalent if there is $g \in \Gamma$ such that $p = qg$. Let \mathcal{A}_{proj} be the subalgebra of \mathcal{A} consisting of all functions which are constant on the equivalence classes of this relation. \mathcal{A}_{proj} is a subset of the center of the algebra \mathcal{A} and thus a commutative algebra (in this case, convolution becomes the usual multiplication). It can be shown that the algebra \mathcal{A}_{proj} is isomorphic with the algebra $C^\infty(M)$ of smooth functions on the space-time manifold M . In this way, we recover the standard space-time geometry and

the standard theory of general relativity (in the Geroch formulation [2]).

Surprisingly, the Γ -part of the above geometry leads to the quantum mechanical approximation. To make the contact with the usual Hilbert space formulation of quantum mechanics let us define the representation $\pi_q: \mathcal{A} \rightarrow \mathcal{B}(\mathcal{H})$ of the algebra \mathcal{A} in the Hilbert space $\mathcal{H} = L^2(G_q)$, $q \in E$, where $\mathcal{B}(\mathcal{H})$ denotes the algebra of bounded operators on \mathcal{H} , by the following formula

$$(\pi_q(a)\psi)(\gamma) = \int_{G_q} a(\gamma_1)\psi(\gamma_1^{-1}\gamma), \quad (1)$$

$a \in \mathcal{A}$, $\psi \in \mathcal{H}$. It is worthwhile to notice that the completion of \mathcal{A} with respect to the norm $\|a\| = \sup_{q \in E} \|\pi_q(a)\|$ is a C^* -algebra. It turns out that if we restrict our analysis to the “wave functions” ψ which are Γ -invariant, i. e., which are constant on the equivalence classes of fibres of G , we essentially obtain quantum mechanics in the Heisenberg picture (see [8, 9]).

We can assume that to every derivation $v \in V$ there corresponds an internal derivation in the image $\pi_q(\mathcal{A})$ given by

$$i\hbar\pi_q(v(a)) = [F_v, \pi_q(a)], \quad (2)$$

for every $a \in \mathcal{A}$, where F_v is a self-adjoint operator (satisfying certain “technical” conditions); the factor $i\hbar$ is added for the future convenience. If $\psi \in L^2(G_q)$ is Γ -invariant, v the usual differentiation (with respect to time in a certain coordinate system), and F_v the Hamiltonian of the system then eq. (2) becomes the usual Schrödinger equation in the Heisenberg picture of quantum mechanics.

3 The EPR Experiment

In the following we shall focus on those nonlocal effects of the noncommutative pre-Planck epoch that can survive the transition to the weak gravity approximation of our model. First, we shall be interested in those effects which make themselves manifest in the quantum mechanical domain. This means that we should consider the subalgebra \mathcal{A}_Γ consisting of those functions on G which are lifting of smooth compactly supported functions on the group Γ , i. e., $\mathcal{A}_\Gamma := \{f \circ pr_\Gamma : f \in C_c^\infty(\Gamma, \mathbf{C})\} \subset \mathcal{A}$, where $pr_\Gamma : G \rightarrow \Gamma$ is the canonical projection. Let $a \in \mathcal{A}_\Gamma$ and $p, q \in E$, $p \neq q$, and let us consider

the following representations of \mathcal{A}_Γ

$$\pi_p(a)(\psi_p) = a_p * \psi_p \quad (3)$$

$$\pi_q(a)(\psi_q) = a_q * \psi_q \quad (4)$$

where $\psi_p \in L^2(G_p)$, $\psi_q \in L^2(G_q)$. G_p and G_q are isomorphic; therefore, ψ_p and ψ_q can be chosen to be isomorphic as well. This, in turn, implies that $\pi_p(a)$ and $\pi_q(a)$ are also isomorphic. Therefore, if $a \in \mathcal{A}_\Gamma$ then its image under the representation π_p does not depend on the choice of the fibre G_p (up to isomorphism). Since $p \in E$ projects down to $\pi_M(p) \in M$, the above can be rephrased by saying that all points of space-time M “know” what happens in the entire fibre G_g , $g \in \Gamma$. The conclusion is that the observational consequences of the proposed model should be looked for among correlations between distant events in space-time rather than among local phenomena (for details see [5]). In the following we shall consider some examples of such correlations.

As the first example we show that the famous Einstein-Podolsky-Rosen effect, experimentally verified by Alain Aspect and others, can be deduced from our model (see [5]). Let us assume that $\Gamma_0 = SU(2)$ is a subgroup of Γ , and let us choose two linearly independent functions on Γ_0 which span the linear space \mathbf{C}^2 . Let further $\hat{S}_z = \pi_p(s)|_{\mathbf{C}^2}$, $s \in \mathcal{A}$, be the z-component of the usual spin operator. By using representation (1) we can write the corresponding eigenvalue equation in the following form

$$\int_{\Gamma_0} s_p(\gamma_1) \psi(\gamma_1^{-1} \gamma) = \pm \frac{\hbar}{2} \psi.$$

Remembering that $s_p = \text{const}$, as one of the solutions of this equation we obtain $\psi = \mathbf{1}_{\Gamma_0}$, and consequently

$$\frac{\hbar}{2} = \pm \int_{\Gamma_0} s_p(\gamma_1).$$

Hence, $(s_p)_1 = +(\hbar/2)(\text{vol}\Gamma_0)^{-1}$, $(s_p)_2 = -(\hbar/2)(\text{vol}\Gamma_0)^{-1}$. The corresponding eigenvalue equations are

$$\pi_p((s_p)_1) \psi = +\frac{\hbar}{2} \psi \quad \text{for } \psi \in \mathbf{C}^+,$$

$$\pi_p((s_p)_2)\psi = -\frac{\hbar}{2}\psi \text{ for } \psi \in \mathbf{C}^-$$

where $\mathbf{C}^+ := \mathbf{C} \times \{0\}$, and $\mathbf{C}^- := \{0\} \times \mathbf{C}$.

Let us consider an observer A who is situated at a point $x_A = \pi_M(p)$, $p \in E$, in space-time M , and an observer B who is situated at a point $x_B = \pi_M(q)$, $q \in E$, in M distant from the point x_A . The observer A measures the z-spin component of the one of the electrons in the EPR experiment, i. e., he acts with the operator $\hat{S}_z \otimes \mathbf{1}|_{\mathbf{C}^2}$ on the state vector $\xi = \frac{1}{\sqrt{2}}(\psi \otimes \phi - \phi \otimes \psi)$ where $\psi \in \mathbf{C}^+$ and $\phi \in \mathbf{C}^-$. Let us assume that the result of the measurement is $+(\hbar/2)$. Therefore, ξ collapses to the state vector $\xi_0 = \frac{\hbar}{\sqrt{2}}(\psi \otimes \phi)$. However, this vector is the same (up to isomorphism) for all fibres G_r , $r \in E$ (let us notice that the analogues of formulae (3) and (4) are also valid for tensor products). In particular, ξ_0 is the same for the fibres G_p and G_q . Now, if the observer B , situated at x_B , measures the z-spin component of the second of the electrons, i. e., if he acts with the operator $\mathbf{1}|_{\mathbf{C}^2} \otimes \hat{S}_z$ on the vector ξ_0 , the only possible result of the measurement could be $-(\hbar/2)$.

In this approach there is no information transfer between A and B , but rather the process of measurement somehow relates to the fundamental level which is atemporal and aspatial.

4 The Horizon Problem

Another example which naturally comes to the mind is the horizon paradox in relativistic cosmology. If the present universe evolved from the pre-Planckian nonlocal stage it is straightforward to expect that various parameters determining its structure at various places are correlated even if these places were never, after the Planck epoch, in any causal contact. In particular, this refers to the “uniformity” of the universe. Let $\rho \in \mathcal{A}$ be an observable corresponding to this “uniformity”; for instance, let ρ be related to the density of the universe. To be an observable ρ must be an Hermitian element of \mathcal{A} , and to leave traces in space-time ρ must be an element of the subalgebra \mathcal{A}_{proj} . The eigenvalue equation of the observable ρ is

$$\int_{G_q} \rho(\gamma_1) \psi(\gamma_1^{-1} \gamma) = r_q \psi(\gamma).$$

Since, in this case, ψ is Γ -invariant it is easy to calculate that

$$r_q = \int_{G_q} \rho(\gamma_1) = \rho(\gamma_1) \text{vol}\Gamma. \quad (5)$$

Let us define the “total phase space” of our system: $L^2(G) := \bigoplus_{q \in E} L^2(G_q)$ with the operator $\pi(\rho) := (\pi_q(\rho))_{q \in E}$ acting on it. Now, eq. (5) assumes the form $r(x) := r_q$, where $\pi_M(q) = x \in M$; it defines the function $r : M \rightarrow \mathbf{R}$ on M . Consequently, if the measurement corresponding to ρ is performed at a point $x \in M$ its result $r(x)$ is correlated with the result $r(y)$ of another measurement of the same quantity at another point $y \in M$ even if these two points are separated by the horizon (in the sense that both $r(x)$ and $r(y)$ are the values of the same function). If one postulates that $r(x)$ is a constant function, it must be proportional to $\text{vol}\Gamma$. In fact, to solve the horizon paradox it is enough to postulate that $r(x)$ is a sufficiently slowly varying function of x (for the sake of simplicity, we do not take into account effects of possible “fluctuations”).

5 The Classical Singularity Problem

Another “global problem” which can be explained by the proposed model is the classical singularity problem in general relativity. For a long time it was known that singularities of stronger types (e. g., curvature singularities) have truly global properties. For instance, the existence of such global features of space-time as the appearance of Cauchy or particle horizons depends on the structure of singularities. They are usually defined as ideal points of space-time or their singular boundaries, and consequently it is more reasonable to speak about singular space-times rather than about singularities in space-time (see [3]). These aspects of the singularity problem suggest that they could somehow be related to the nonlocal physics of the pre-Planckian era. This indeed turns out to be the case.

Let us construct the generalized fibre bundle of orthonormal frames on a space-time M with its singular boundary ∂M (for instance Schmidt’s b-boundary), $\bar{M} = M \cup \partial M$. Let E be the total space of this generalized fibre bundle. The fibres of E over ∂M are degenerate (in the case of the closed Friedman universe and Schwarzschild’s solution with their b-boundaries these fibres degenerate to the single points). However, it can be shown that if we

construct the groupoid $G = E \times \Gamma$, the fibres of G over degenerate fibres of E are regular (for details see [4]). This procedure truly deserves the name “desingularization process”. Now, we can define the algebra \mathcal{A} of smooth, complex valued, compactly supported functions on G , and proceed exactly in the same way as above. In the noncommutative regime there are no points, but there are states of the system (represented by the states on the algebra \mathcal{A} , i. e., by positive, suitably normed functionals on \mathcal{A}), however with no possibility to distinguish between singular and non-singular states [4, 6]. In these circumstances it is meaningless to speak about singularities in any sense.

If we reverse this construction, starting from the non-commutative geometry of the groupoid G and going down to the space-time manifold, we can clearly see that singularities arise in the process of taking the double quotient by the action of the group Γ : first to obtain $E = G/\Gamma$, and then to obtain the space-time with singularities $\bar{M} = E/\Gamma$ (this is studied in [6])

The question: will the future theory of quantum gravity remove singularities from our picture of the universe? is usually presupposed to admit two answers: the answer “yes” has become a kind of common wisdom; the answer “no” is adopted only by a few. The above results open the third possibility. On the fundamental level (below the Planck threshold) it is meaningless even to ask about singularities. The noncommutative geometry shaping this level is totally nonlocal: there is no space, no time (in their usual sense) and no distinction between singular and non-singular states. In spite of this, the true (albeit generalized) dynamics is possible (see [7]). It is only in the transition process from the noncommutative regime to the commutative geometry (the sort of the first phase transition), when the space-time emerges and its singular boundaries are produced. From the “point of view” of the fundamental level there are no singularities; from our point of view, who are macroscopic observers, there was the singularity in the beginning of our universe, and possibly there will be one at its end.

References

- [1] Connes, A., *Noncommutative Geometry*, Academic Press, New York - London, 1994.

- [2] Geroch, R., *Commun. Math. Phys.* **26**, 1972, 271.
- [3] Hawking, S. W. and Ellis, G. F. R., *The Large Scale Structure of Space-Time*, Cambridge University Press, Cambridge, 1973.
- [4] Heller, M. and Sasin, W., *J. Math. Phys.* **37**, 1996, 5665.
- [5] Heller, M. and Sasin, W., in: *Particles, Fields and Gravitation*, ed. J. Rembieliński, The American Institute of Physics, Woodbury - New York, 1998, 234.
- [6] Heller, M. and Sasin, W., “Origin of Classical Singularities”, *Gen. Rel. Gravit.* in press, preprint gr-qc/9812047.
- [7] Heller, M. and Sasin, W., *Phys. Lett. A* **250**, 1998, 48.
- [8] Heller, M. and Sasin, W., “Noncommutative Unification of General Relativity and Quantum Mechanics”, *Int. J. Theor. Phys.*, in press.
- [9] Heller, M., Sasin, W. and Demaret, D., *J. Math. Phys.* **38**, 1997, 5840.
- [10] Landi, G., *An Introduction to Noncommutative Spaces and Their Geometries*, Springer, Berlin - Heidelberg, 1997.
- [11] Madore, J. and Mourad, J., *J. Math. Phys.* **39**, 1998, 423.